



2.3.2 Vector addition

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Introduction

- In this topic, we will
 - Describe the operation of vector addition
 - First look at real vector addition
 - Next look at complex vector addition
 - Discuss some properties





Real vector addition

- Given two vector in $\mathbf{R}^n$, we can add the corresponding entries
 - This is usually written as $\mathbf{u} + \mathbf{v}$

- For example, if $\mathbf{u} = \begin{pmatrix} 2.7 \\ 0.8 \\ -3.9 \\ 2.8 \\ -7.5 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1.2 \\ -3.7 \\ 6.4 \\ 0.8 \\ 9.5 \end{pmatrix}$,

$$\text{then } \mathbf{u} + \mathbf{v} = \begin{pmatrix} 2.7 \\ 0.8 \\ -3.9 \\ 2.8 \\ -7.5 \end{pmatrix} + \begin{pmatrix} 1.2 \\ -3.7 \\ 6.4 \\ 0.8 \\ 9.5 \end{pmatrix} = \begin{pmatrix} 3.9 \\ -2.9 \\ 2.5 \\ 3.6 \\ 2.0 \end{pmatrix}$$





Real vector addition

- We may also just write

$$\begin{pmatrix} 6.8 \\ 1.9 \\ 2.8 \end{pmatrix} + \begin{pmatrix} 8.5 \\ 6.3 \\ -4.9 \end{pmatrix} = \begin{pmatrix} 15.3 \\ 8.2 \\ -2.1 \end{pmatrix}$$

$$\mathbf{u} + \mathbf{v} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{pmatrix} = \begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \\ u_3 + v_3 \\ u_4 + v_4 \\ u_5 + v_5 \\ u_6 + v_6 \end{pmatrix}$$





Real vector addition

- Note that,

$$\mathbf{0}_4 + \mathbf{v} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \mathbf{v} \quad \mathbf{v} + \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} 2v_1 \\ 2v_2 \\ 2v_3 \\ 2v_4 \end{pmatrix} = 2\mathbf{v}$$

- Also, if $\mathbf{v} = \begin{pmatrix} 1.2 \\ -3.7 \\ 6.4 \\ 0.8 \\ 9.5 \end{pmatrix}$, $-1 \cdot \mathbf{v} + \mathbf{v} = \begin{pmatrix} -1.2 \\ 3.7 \\ -6.4 \\ -0.8 \\ -9.5 \end{pmatrix} + \begin{pmatrix} 1.2 \\ -3.7 \\ 6.4 \\ 0.8 \\ 9.5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \mathbf{0}_5$





Real vector addition

- Also note that

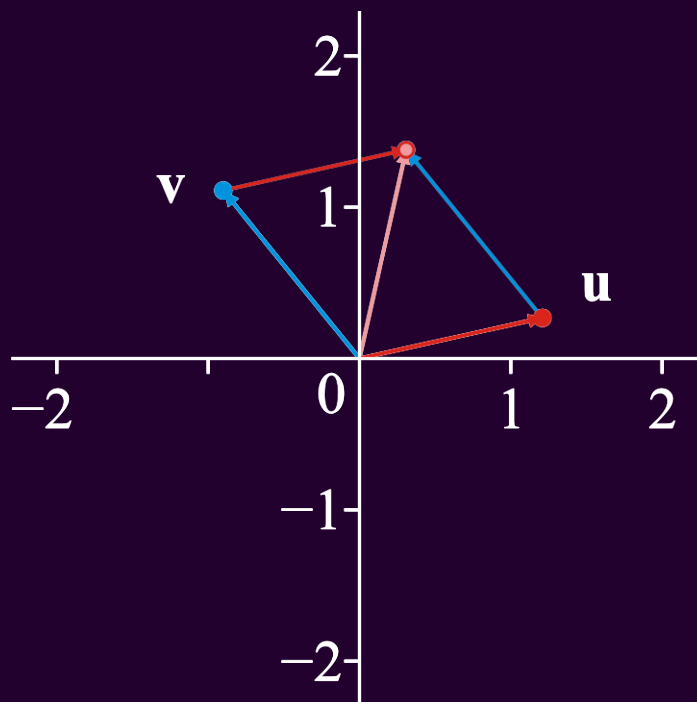
$$\mathbf{u} + \mathbf{v} = \begin{pmatrix} 2.1 \\ 5.9 \\ 6.1 \\ 0.2 \\ -3.6 \\ 7.4 \\ 1.8 \end{pmatrix} + \begin{pmatrix} -4.0 \\ 1.3 \\ -3.9 \\ 8.1 \\ 1.5 \\ 2.9 \\ 0.8 \end{pmatrix} = \begin{pmatrix} -1.9 \\ 7.2 \\ 2.2 \\ 8.3 \\ -2.1 \\ 10.3 \\ 2.6 \end{pmatrix} = \begin{pmatrix} -4.0 \\ 1.3 \\ -3.9 \\ 8.1 \\ 1.5 \\ 2.9 \\ 0.8 \end{pmatrix} + \begin{pmatrix} 2.1 \\ 5.9 \\ 6.1 \\ 0.2 \\ -3.6 \\ 7.4 \\ 1.8 \end{pmatrix} = \mathbf{v} + \mathbf{u}$$





Geometric interpretation

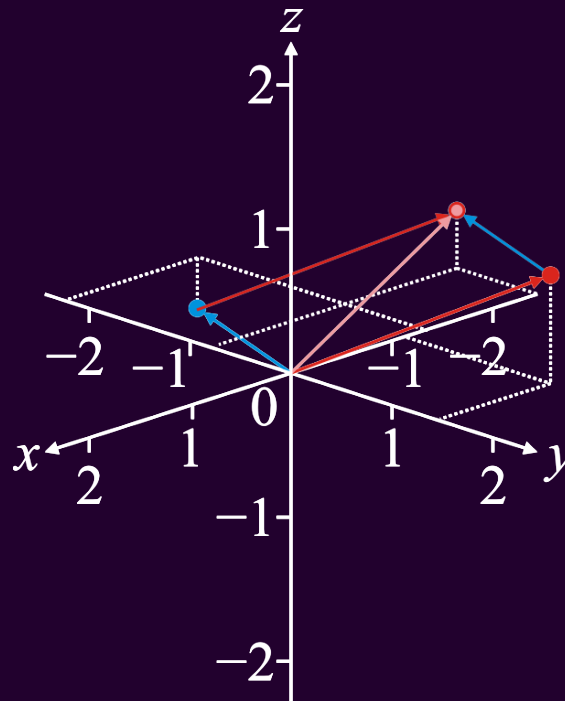
- In two dimensions ($\mathbf{R}^2$), if vectors are interpreted as *arrows* from the origin,
the sum of two vectors is equivalent to placing the base of one vector at the head of the other





Geometric interpretation

- A similar interpretation is appropriate for 3-d vectors





Complex vector addition

- Given two vector in $\mathbf{C}^n$, we can add them in a similar manner:

- For example, if $\mathbf{u} = \begin{pmatrix} 3.9 + 5.7j \\ 1.8 - 8.6j \\ -0.5 + 3.4j \\ 1.9 - 3.1j \\ -3.9 - 8.1j \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1.2 + 0.3j \\ -3.7 - 1.5j \\ 6.4 + 0.0j \\ 0.8 - 8.2j \\ 9.5 + 1.5j \end{pmatrix}$

$$\text{then } \mathbf{u} + \mathbf{v} = \begin{pmatrix} 5.1 + 6.0j \\ -1.9 - 10.1j \\ 5.9 + 3.4j \\ 2.7 - 11.3j \\ 5.6 - 6.6j \end{pmatrix}$$





Complex vector addition

- Note that if $\mathbf{v} \in \mathbf{C}^n$, then it is still true that

$$\mathbf{0}_n + \mathbf{v} = \mathbf{v} \quad \text{and} \quad -1 \cdot \mathbf{v} + \mathbf{v} = \mathbf{0}_n$$

- Also, we expect that $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ and $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$





Observations

- Note that:
 - Adding two n -dimensional vectors results in an n -dimensional vector
 - We are either dealing with real vectors or complex vectors
 - There is no point in asking “What happens if you add a real vector to a complex vector?”
 - It’s like asking “What happens if you start with five cups of water and someone gives you $1 + 2j$ more cups?”
 - Similarly, we only add vectors of the same dimension
 - Adding a thousand-dimensional vector to a 2-dimensional vector is meaningless
 - You may be able to interpret a 2-d vector as a 3-d vector, and then add that 3-d vector onto another 3-d vector





Summary

- Following this topic, you now
 - Understand the concept of vector addition
 - Have seen examples for both real and complex numbers
 - Looked at how vector addition interacts with the zero vector
 - Looked at some properties and made some observations





References

- [1] https://en.wikipedia.org/wiki/Scalar_multiplication





Acknowledgments

None so far.





Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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for more information.





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